

CC-5 Sem-3 Linear Algebra

1. Define linear independence of a set of vectors. Show that every subset of a linearly independent set of vectors is also linearly independent.
2. If A and C be square matrices, show that

$$\begin{vmatrix} A & B \\ 0 & C \end{vmatrix} = |A| |C|$$

3. Show that basis of a vector space is not unique.
4. Find A^{-1} where $A_{p \times p} = ((a_{ij}))$; $a_{ij} = \rho$ if $i \neq j$ and $a_{ii} = 1$.
5. Consider the 1×6 vectors

$$\alpha_1 = (1 \ 1 \ 0 \ 1 \ 0 \ 0) \quad \alpha_2 = (1 \ 1 \ 0 \ 0 \ 1 \ 0) \quad \alpha_3 = (1 \ 1 \ 0 \ 0 \ 0 \ 1)$$

$$\alpha_4 = (1 \ 0 \ 1 \ 1 \ 0 \ 0) \quad \alpha_5 = (1 \ 0 \ 1 \ 0 \ 1 \ 0) \quad \alpha_6 = (1 \ 0 \ 1 \ 0 \ 0 \ 1).$$

Find the dimension of the vector space generated by the above vectors. Also find a vector which is orthogonal to the vector space spanned by $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$ and α_6 .

6. State and prove a set of necessary and sufficient conditions for positive definiteness of a quadratic form.
7. Let A be a $p \times p$ symmetric matrix. Show that there exists a matrix C of order $p \times p$ such that $C'AC=I$.
8. Explain the concept of basis of a vector space. Show that choice of basis is not unique.
9. Let $A^{n \times n}$ be a square matrix such that $A = -A'$ and $(I + A)$ is non-singular, where I is the identity matrix of order n. Then show that $(I - A)(I + A)^{-1}$ is an orthogonal matrix.
10. For any two square matrices A and B of order n show that $\text{Rank}(AB) \geq \text{Rank}(A) + \text{Rank}(B) - n$.
11. Define a vector space. Show that choice of basis is not unique but the number of vectors in a basis of a vector space is constant.

12. Show that a symmetric matrix A is positive definite if there exists a non-singular matrix P such that $A = P'P$.
13. Show that if the quadratic form $X'AX$ is positive definite and A^{-1} exists, then $X'A^{-1}X$ is also positive definite.
14. Define the basis of a vector space. Describe the Gram-Schmidt orthogonalization process.
15. Define a skew-symmetric matrix. Show that the determinant of every skew-symmetric matrix of odd order is zero and that of even order is a perfect square.
16. Find the inverse of the following matrix of order $n \times n$

$$\begin{pmatrix} a & b & b & \dots & b \\ b & a & b & \dots & b \\ b & b & a & \dots & b \\ \dots & \dots & \dots & \dots & \dots \\ b & b & b & \dots & a \end{pmatrix}.$$

17. Give the definition of a vector space and subspace. If V_1 and V_2 are two subspaces of a vector space, say V , then show that $V_1 \cap V_2$ is also a subspace of V , although $V_1 \cup V_2$ may not be the same.
18. What is a basis of a subspace? Define an orthogonal basis of a vector space. Starting from an arbitrary basis, how can one find an orthogonal basis?
19. Evaluate:

$$\Delta_k = \begin{vmatrix} np_1q_1 & -np_1p_2 & -np_1p_3 & \dots & -np_1p_k \\ -np_2p_1 & np_2q_2 & -np_2p_3 & \dots & -np_2p_k \\ \dots & \dots & \dots & \dots & \dots \\ -np_kp_1 & -np_kp_2 & -np_kp_3 & \dots & np_kq_k \end{vmatrix}$$

where $q_i + p_i = 1$ for $i = 1$ (1) k and $\sum_{i=1}^k p_i = 1$.

20. Let A and B be two matrices such that B is obtained from A by interchanging its first and second rows. Find a non-singular matrix P such that $PA = B$.

21. Find the angle between two non-zero vectors. Hence derive the Cauchy-Schwartz inequality.
22. For a square matrix A , suppose $A\mathbf{x} = \mathbf{0}$ for some non-zero vector \mathbf{x} . Explain why A is singular.
23. What is the echelon form of a matrix? Prove that the rank of a matrix is equal to the number of non-zero rows in its echelon form.
24. Prove that for any two square matrix $A^{n \times n}$ and $B^{n \times n}$, $|AB| = |A| |B|$.
25. Define rank of a matrix. Show that, in a matrix, the number of linearly independent rows is same as the number of linearly independent columns.
26. Define a positive definite (p.d.) matrix. If $\delta_{ij} = 1$ or 0 as $i =$ or $\neq j$, show that the matrix $((\delta_{ij} + x_i x_j))$, $i, j = 1, 2, \dots, p$ is p.d. Stating the necessary results, show that the characteristic roots of a p.d. matrix are all positive. Hence establish that the sum of the diagonal elements of a p.d. matrix is positive.
27. Let A and B be two matrices of orders p by q and q by p , respectively. Show that $|AB| = 0$ if $p > q$.
28. Show that all the vectors $(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3)$ in a vector space \mathbf{V}_3 which obey $x_1 - x_3 = x_2$ form a subspace \mathbf{V} . Then show that \mathbf{V} is spanned by $\mathbf{a}_1 = (1, 0, 1)$ and $\mathbf{a}_2 = (0, 1, -1)$. Find an orthonormal basis of \mathbf{V} . Also find a vector which is orthogonal to all vectors in \mathbf{V} .
29. Consider the vectors $\mathbf{X}_1 = (1, 3, 2)$ and $\mathbf{X}_2 = (-2, 4, 3)$ in \mathbf{R}^3 . Show that the set spanned by \mathbf{X}_1 and \mathbf{X}_2 is given by

$$\{ (\xi_1, \xi_2, \xi_3) : \xi_1 - 7\xi_2 + 10\xi_3 = 0 \}$$

30. Define rank of a matrix. If A' be the transpose of a matrix A , show that
- $$\text{rank}(A'A) = \text{rank}(AA') = \text{rank}(A).$$

CC-6 Sem-3 Demography and Vital Statistics

1. Discuss rate and ratio of vital events with examples.
2. Discuss the errors in census data in vital statistics.
3. Discuss the errors in registration data in vital statistics.
4. Define specific death rate (SDR) and explain the merits of the measure.
5. What is the method of standardization as a measure of mortality?
6. What is indirect method of standardization?
7. “Mortality very much depends on the compositions of the population, like – age, sex, occupation, place of dwelling etc.” – Explain the statement with appropriate examples.
8. Discuss the concept of Neonatal and Perinatal Mortality Rates.
9. Define infant mortality rate and discuss its drawback.
10. What is life table? Establish an approximate relation between e_x and e_x^0 by defining them properly.
11. Interpret L_x column of a life table. Discuss the formula to calculate the same for last age interval of a life table.
12. Construct a complete life table based on registration data only.
13. Define the life table function e_x^0 . Show that, under suitable assumption,

$$e_x^0 = 1 - \frac{1}{2}q_x + \frac{1}{l_x} \sum_{i=1}^{\infty} l_{x+i} - \frac{1}{2l_x} \sum_{i=1}^{\infty} l_{x+i} q_{x+i}$$

$$e_x^0 = \frac{1}{2} + \frac{1}{l_x} \sum_{i=1}^{\infty} i d_{x+i}.$$

14. Distinguish between complete and abridged life table. Discuss a form of abridged life table.
15. Starting from crude birth rate, describe the step by step modification applied to find the best possible measure of fertility rate.

16. Discuss indirect method of standardization. How can you compute q_x value at $x = 0$ using registration data only? Starting from a suitable form of l_x derive the approximate formula $L_x = \frac{1}{2}(l_x + l_{x+1})$ and $L_x = -\frac{d_x}{\ln p_x}$.

17. Prove that, under linearity assumption of l_x function,

$$e_x^0 = 1 - \frac{1}{2}q_x + \frac{1}{l_x} \sum_{i=1}^{\infty} l_{x+i} - \frac{1}{2l_x} \sum_{i=1}^{\infty} l_{x+i}q_{x+i}$$

18. Show that ${}_n p_x = \frac{(e_x e_{x+1} \cdots e_{x+n-1})}{(1+e_{x+1})(1+e_{x+2}) \cdots (1+e_{x+n})}$ and ${}_n p_x = \prod_{i=0}^{n-1} \left(\frac{e_{x+i}^0 - \frac{1}{2}q_{x+i}}{1+e_{x+1+i}^0} \right)$

19. Compare gross reproduction rate (GRR) and net reproduction rate (NRR) as a measure of population growth.

20. "GRR serves as an upper bound of NRR" – Justify the statement.

21. Discuss the fitting of logistic curve and its properties.

22. Let P_1, P_2 and P_3 be the population at three equidistant point of time – then the ultimate population L of the logistic curve is given by $(1/P_1 + 1/P_3 - 2/P_2) / (1/P_1 P_3 - 1/P_2^2)$.

23. How do you estimate population size of a region (or country) by component method?

24. What is balance equation? Discuss how we can use it to check the consistency of registration data.

25. What is Morbidity Incidence and Morbidity Prevalence Rates?

CC-7 Sem-3 Statistical Computing and Numerical Analysis Using C

1. What do you mean by numerical differentiation? Use Lagrange's interpolation formula to derive a formula for numerical differentiation.
2. Derive the Newton-Raphson method for solving an equation involving single unknown and discuss its geometric significance.
3. Prove that the r^{th} difference of a polynomial of the n^{th} degree is a polynomial of the $(n - r)^{\text{th}}$ degree if $r < n$. What happens when (i) $r = n$, (ii) $r > n$?
4. Given U_0, U_1, U_2, U_3, U_4 and U_5 , prove (stating necessary assumptions) that

$$U_{\frac{1}{2}} = \frac{1}{2}C + \frac{25(C - B) + 3(A - C)}{256}$$

where $A = U_0 + U_5, B = U_1 + U_4, C = U_2 + U_3$.

5. Show that Lagrange's interpolation formula for the second degree curve passing through the three points $(X = i, Y = y_i), i = -1, 0, 1$ can be expressed as

$$y = C_{-1}(x)y_{-1} + C_0(x)y_0 + C_1(x)y_1,$$

where $C_i(x)$ is a quadratic in x . Obtain $C_i(x)$ explicitly in terms of x for $x = -1, 0, 1$.

6. Derive the general quadrature formula and hence derive the Simpson's 1/3-rd rule.
7. Establish an operational identity among the operators E, Δ and D .
8. Describe an evaluating method when one value of $f(x)$ corresponding to the equidistant values of x is missing.
9. Assume that $y = f(x)$ has an extremum in the interval $(-1, 1)$. Show that Newton's forward interpolation formula upto second differences would provide an approximate value of the extremum value equal to

$$f(0) - \frac{1}{8} \left[\frac{\{f(1) - f(-1)\}^2}{f(1) - 2f(0) + f(-1)} \right].$$

In what way would you be sure of this value to be a maximum?

10. Suppose $(x_0, y_0), (x_1, y_1), \dots, (x_n, y_n)$ are $(n + 1)$ pairs of values of the argument x and entry $y = f(x)$. Then show that the polynomial used for Lagrange's interpolation formula can be represented in the form

$$\sum_{j=0}^n \frac{Q(x)}{(x - x_j) Q'(x_j)} y_j$$

where $Q(x) = \prod_{j=0}^n (x - x_j)$ and $Q'(x_j) = \frac{d}{dx} Q(x) \Big|_{x=x_j}$. Explain a property of the coefficients of y_j which can be used as computational check.

11. The first difference of a function $\log_e U_x$ are in arithmetic progression. Evaluate the function U_x .

12. Write a note on 'convergence of the iteration method to compute a real root of an equation in one unknown'.

13. For a second degree polynomial U_x , show that $\int_1^3 U_x dx = 2U_2 + \frac{1}{12}(U_0 - 2U_2 + U_4)$.

14. Write a program in C to prepare a frequency distribution taking class intervals of length 5 for the

following data:

79, 65, 59, 42, 57, 45, 68, 47, 63, 53, 30, 35, 15, 48, 26, 20, 40, 55, 50, 57, 59, 63, 77, 77.

Your output should contain the class intervals, corresponding frequencies and the corresponding relative frequencies.

15. Write a program in C to check whether a given square matrix of order m is skew symmetric or not.

16. Write a function in C that will return the values of $g(x) = x \cos^{-1} x - \log_2(1 + x)$; $0 \leq x \leq 1$ when referenced in the main function. Now, within the main function calculate 200 values of $g(x)$ corresponding to 200 equally spaced values of x and hence find the approximate area of the region enclosed by $y = g(x)$, $x = 0$ and $x = 1$.

17. Nine students were interviewed by two judges and their scores are given below:

Judge 1:	6	11	10	15	8	7	9	14	12
Judge 2 :	8	10	13	9	12	15	7	6	18

Write a program in C to compute Pearson's product moment Correlation Coefficient with the data given above and comment.

18. Write down a program in C to compute the Correlation Co-efficient of two variables 'X' and 'Y' and the Linear Regression Co-efficient of 'Y' on 'X' given 'n' pairs of values.

19. Write a program in C to calculate Mean, Standard Deviation, Skewness and Kurtosis of the following dataset on the weekly wages (in rupees) of 28 labourers working in a factory.

220, 268, 258, 242, 210, 268, 272, 242, 311, 290, 300, 320, 319, 304, 302, 318, 306, 292, 254, 278, 210, 240, 280, 316, 306, 215, 256, 236

20. Write a program in C to compute the proportion of even numbers among the first 60 natural integers.

21. Given that $f(0) = 1, f(1) = 3, f(3) = 55$, find the Lagrange polynomial of appropriate degree, which fits the given data.
22. Write a program in C to find the proportion of the students with marks in the interval $[50, 70]$, when the marks of 20 students in a class are provided.
23. Write a C program to find the median of any given set of 15 numbers using a function with the data array as argument.
24. Find the iterative methods based on the Newton-Raphson method for finding e^N , where N is a positive real number.
25. Write a C function to calculate the mean of n (>100) numbers used for looping structure, where n is not specified.
26. Describe how Lagrange's interpolation formula can be used to find the approximate root of a given equation.
27. Prove that $f(4)$ can be expressed as $f(3) + \Delta f(2) + \Delta^2 f(1) + \Delta^3 f(1)$.
28. Write a program in C to determine the inverse of a non-singular matrix.
29. Write a program in C to determine the rank of a $m \times n$ matrix.
30. Write a program in C to check whether a matrix is non-singular or not.

SEC-A-1 Statistical Data Analysis Using R

1. Short questions :

- (a) Define open source software.
- (b) How will you search R for all commands containing the term 'test'? (c) What will be the value of $\text{atan}(1)$?
- (d) Using an example, distinguish between $\sin(x)$ and $\sin\pi(x)$.
- (e) Write down the output for "Hello" > "Good bye". Justify your answer.
- (f) Why do you use `getwd()`?
- (g) Give the syntax to create a bit of 2 variables in R.
- (h) Suppose, `day <- c('Mon', 'Tue', 'Wed', 'Thu')`. Find the output for `day[-3]`. Explain your answer.
- (i) Explain how to use 'rep' to get the output `c(1,2,3,1,2,3)`.
- (j) Suppose, `x <- c(2,3,5)` and `y <- c(0,3,6)`. Write down the output of `x*y` and explain.
- (k) What is the use of 'header' in read table?
- (l) Write a code to generate the matrix

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

- (m) Give an example showing the application of 'rbind'.
- (n) Write a code to get the value of π rounded upto 5 decimal places. (o) Write a code to find out the eigen vectors of a matrix.
- (p) What code will you use to check the list of commands in the MASS library? (q) Create a factor variable with levels 2, 4 and 6.
- (r) What is the function of `type = 'l'` in R graphics?

2 . Long questions :

- (a) How will you fit a simple linear regression in R on a categorical covariate? How will you extract the regression coefficients from the summary?
- (b) Suppose the square root value of 32 is 5.6568. Write down the outputs of (i) `sqrt(-32)`, (ii) `sqrt(-32 + 0i)`, (iii) `sqrt(as.complex(-32))`.
- (c) Suppose `x <- c(1,2,3,4)`. What will be the output of
(i) `1/x`, (ii) `c(x, 0, x)`, (iii) `x % / % 2`?
- (d) Give the R code using 'Seq' to generate the vectors
(i) `(-2, -1, 0, 1, 2)` (ii) `(3, 3.5, 4, 4.5, 5)` (iii) `(3,....., 5)` of length 100 with equispaced inputs between 3 and 5.
- (e) Give an example of the application of 'abline' in R graphics. How will you superimpose the regression line on a scatterplot using 'abline'?
- (f) Consider `u <- c(3, 5, 7, 5, 3, 2, 8, 5, 6, 9, 8, 8)`. Write codes to find
(i) length of the vector, (ii) unique elements (iii) frequency distribution.
- (g) The following table gives the actual expenditure in the public sectors during the 4th five year plan.

Write a code to draw a pie chart for the data.

Sectors	Expenditure (₹ in crores)
Agriculture	2217
Power	2085
Large Industry	3090
Small Industry	295
Communication	3173
Miscellaneous	2575

- (h) How will you generate a sample of size 100 from the standard normal distribution and then find the quantiles of the generated data?

- (i) Give an example of how to use the 'byrow' option of two vectors. Write the codes for (i) matrix multiplication (ii) element wise multiplication (iii) cross product of two vectors (iv) solution to a linear system $Ax = b$.
- (j) An inbuilt dataset 'women' gives heights and weights for American women aged 30–39 in the year 1975. (i) Write code(s) to get the names of the columns and the first few rows of the dataset along with the header. (ii) Write code to change the inbuilt variable names to VI and VI (iii) Compute the median of height, variance of weight and correlation coefficient between the two variables. (iv) Write R codes to setup a linear regression model taking 'height' as the independent variable and estimate the regression coefficients.
- (k) (i) The following table gives the number of students in the colleges of Kolkata in the year 2001 according to their birth places. Write a code to graphically represent this data.

Place of birth	No. of students
Kolkata	22400
Outside Kolkata but within W.B.	14600
Outside W.B.	6000

Add a legend to the plot

- (ii) A typical standard normal table has a column with x values and another with $\Phi(x)$ values. Write an R code to generate the table. What code will you write to get the lower tail C.D.F. value for the probability 0.5 and d.f. = 3 for a t-distribution?

SEC-A-2 Sem-3 Research Methodology

- 1) What is the meaning of Research? What is a Research problem?
- 2) What is the meaning of Research Design? Mention two features of a good research design.
- 3) Differentiate between qualitative and quantitative approach of research, Longitudinal, survey & experimental Research.
- 4) Differentiate between Research Methods & Research Methodology.
- 5) Write the classification of measurement scales. Define any one of the widely used scales of measurement.
- 6) What are Reliability and Validity in the context of measurement?
- 7) What is Practicality Characteristic of a measuring instrument? What is Characteristic of Accuracy of a measuring scale?
- 8) Write your concept on the following:
 - i) “control” in the context of Research Designing
 - ii) “Confound Relationship” in the context of Designing a Research
 - iii) Extraneous or Confounding Variable
 - iv) Control Group in an experimental hypothesis testing research
 - v) Experimental Group in an experimental hypothesis testing research
 - vi) Treatment and experimental unit
 - vii) Research Hypothesis
 - viii) Experimental & Non-experimental hypothesis testing research
 - ix) Research Process
 - x) Sampling Frame and Coverage Error
- 9) Give two examples to describe the role of Research
- 10) Show the “Research Process” in a flow chart
- 11) What are the different types Research?
- 12) What does “Research Technique” mean?
- 13) Mention different Research Designs.
- 14) What is a Questionnaire? What are the advantages and disadvantages of Questionnaire?